What you need to know about logarithms
266: Financial Markets and Institutions

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▶ ...

- We will make some use of natural logarithms in this class
- If this is foreign to you, you should read up a bit
- But we will use only a couple of facts

▶ **Aside:: Wikipedia**

- The wikipedia page is pretty decent on natural logaritms.

  go

  http://en.wikipedia.org/wiki/Natural%5Flogarithm

▶ **Natural logarithm**

- The 'natural log' of x is more formally called the logarithm in base e of x
- The natural log of x is the number, z, such that $e^z = x$

  where $e \approx 2.71828$

- Often written, $\log_e(x)$ or $\ln(x)$ or sometimes simply $\log(x)$

  but this latter might be confused with $\log_{10}(x)$.

▶ **Simple algebra implies**

- $\ln(a \times b) = \ln(a) + \ln(b)$
- $\ln(a^c) = c \ln(a)$
- Thus, $\ln(a^c b) = c \ln(a) + \ln(b)$

▶ **A useful approximation:**

- For small z:

$$\ln(1+z) \approx z$$

- (Note: this is a first order Taylor series approx.)

▶ **Approximation: How small?**

z	$\ln(1+z)$
0.03	0.0295
0.05	0.049
0.10	0.095
0.20	0.182
1×10^6	14

- Typical levels of interest rates in stable economies are, say, between 0 and 12 percent.

  That is i between 0 and 0.12

- In this range the approximation $\ln(1+i) \approx i$ is pretty good.
- For large values, not so good

▶ **Useful?**

- This approximation is useful because we have a lot of expressions of the form, e.g.,

$$(1+i)^t$$

- Taking logs and using the approximation, this becomes the simpler expression:

$$\begin{aligned} \ln\left((1+i)^t\right) &= t\ln(1+i) \\ &\approx ti \end{aligned}$$

▶ **...** Approximations like this are used heavily every day in financial markets