Term structure of interest rates
266: Financial Markets and Institutions

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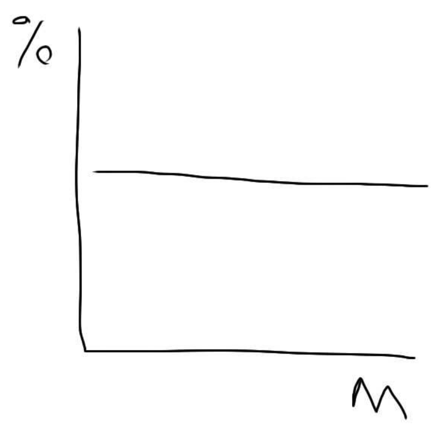
- **Yield curve**
  - On any given day, the market interest rates for IOUs/loans of different maturities is called the term structure of interest rates
  - When we plot these rates against the term or maturity of the loan we call this the yield curve.
- **When we use same i for each horizon:**



- **In practice,**
  - The yield curve is usually sloped at least slightly upward and moves around a great deal
- **Let's explore the history of the yield curve**

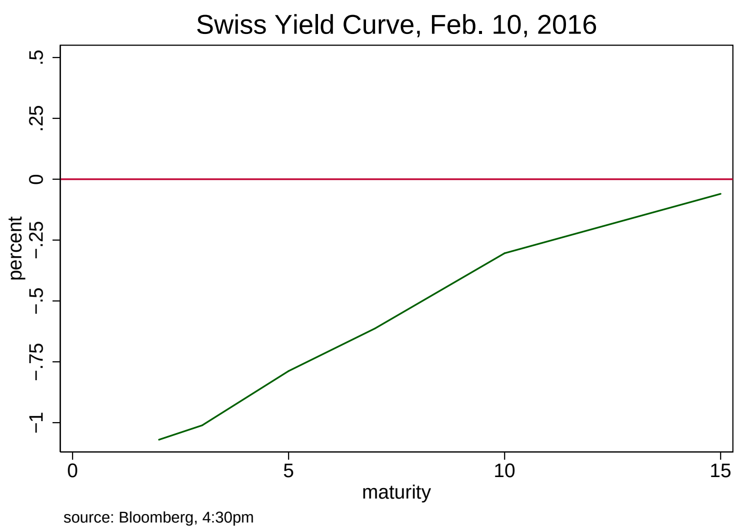
- ...
  - You can download this exciting movie from the course website

    mp4: go

    http://e105.org/e266/download/yc.mp4

- **A few things to notice**
  - Usually (but not always) a bit upward sloping
  - Negatively sloped when rates peaked around 1980
  - Term structure is very low right now

- **But not as low as, e.g., in Switzerland**



- **Bernanke**
  - In 2013, Chm. Bernanke gave a nice speech about long-term rates and why they are so low

    Worth a read: go

    http://federalreserve.gov/newsevents/speech/bernanke20130301a.htm

  - It's a bit ominous that 3 years later rates in much of the world are even lower

- **A great deal of what goes on in financial markets depends on understanding the term structure of interest rates.**

- **Notation:**

- Let's define i_{ht} to be the annualized interest rate on day t on an h-year loan or zero coupon bond

  single payment coming in h years.

▶ **Present value**

- With constant interest, the present value of a cash flow, $s_{t+1}, \ldots, s_{t+M}$, is

$$PV_t = \sum_{j=1}^{M} \frac{s_{t+j}}{(1+i)^j}$$

- When interest rates vary by horizon:

$$PV_t = \sum_{j=1}^{M} \frac{s_{t+j}}{(1+i_{jt})^j}$$

- We have to use the j-period interest rate in discounting the payment j periods in future.

  Still raised to j^{th} power b/c i_{jt} is stated at an annual rate.

▶ **How are rates of different maturities related?**

▶ **Theories of the term structure**

- We'll discuss some economic theories of term structure behavior
- To do so, it is useful to step back and discuss the law of one price (LOOP)

▶ **Aside:: LOOP**

- What is the law of one price (LOOP) in economics?

  Equivalent items must sell for the same price under the conditions of perfect competition

- Suppose A and B are identical, give the argument why the market will tend to push the prices to the same level.

  Demand shifts from more expensive one to cheaper one, driving one price down and the other up.

  Similarly, supply may shift as well, driving the prices together.

▶ **Aside:: LOOP and hedge fund strategies**

- 'LOOP thinking' is at the center of many hedge fund investment strategies
- Key intuition: you detect two nearly identical streams that sell for different prices, and do some version of buying the cheap one and sell the expensive one.

- Actually, the investments often look like a bet that the deviation from LOOP won't last

    That is you bet that prices will converge to some price consistent with LOOP. Sometimes call 'convergence strategies.'

▶ LOOP in finance

- Much of finance theory starts with applying loop under special assumptions

    For example, assume away uncertainty…

- Much of modern rocket science in finance is based in sophisticated application of LOOP

    Showing that two investment strategies that may look quite different are actually equivalent and should thereby earn the same return

▶ LOOP+certainty and the term structure

- For a moment ignore all uncertainty.
- Suppose you want to borrow or invest for 10 years.
- In our notation, there 10-year interest rate at time t is denoted $i_{10,t}$
- If you invest $1, at the end of 10 years you will have

$$(1 + i_{10,t})^{10}$$

▶ An equivalent alternative investment

- I could invest $1 for 3 years and then roll over the proceeds into a new 7 year loan.
- At the end of 3 years, I'll have: $(1 + i_{3,t})^3$
- At the end of 10 years, I'll have

$$(1 + i_{3,t})^3 (1 + i_{7,t+3})^7$$

- Setting aside uncertainty, the LOOP says that these two ways to invest for 10 years must return the same amount.
- Why

    If, say the single 10-year loan returned more, folks would shift funds out of the 3+7 alternative into the 10. This drives up the price of the 10 (driving down it's return) and drives down the price of the alternative (driving up its return). This ends when the two are equal.

▶ Thus,

- Loop+certainty imply

$$(1 + i_{10,t})^{10} = (1 + i_{3,t})^3 (1 + i_{7,t+3})^7$$

- That is, market forces would drive these 3 interest rates to a point where this equation holds

- **Consider 10-year vs. rolling over 10 1-year bonds**
  - LOOP says:
  $$(1 + i_{10,t})^{10} = (1 + i_{1,t}) \times (1 + i_{1,t+1}) \times \ldots \times (1 + i_{1,t+9})$$
  - Take natural log:
  $$10 \ln(1 + i_{10,t}) = \sum_{j=0}^{9} \ln(1 + i_{1,t+j})$$
  - Apply $\ln(1 + z) \approx z$ for small z:
  $$i_{10,t} \approx \frac{1}{10} \sum_{j=0}^{9} i_{1,t+j}$$
  - Under this story, the 10-year rate is approximately equal to the average of the 10, 1-year rates that will prevail over the next 10 years.
  - Setting aside uncertainty , LOOP gives us a simple way to relate long-term rates to the future short-term rates that will prevail.

- **Question**
  - If the current 10-year rate is higher than the current 1-year rate, then in this world we know that the 1-year rate in the future will have to be higher.

    The long rate is the average of the future short rates, so we have to pull the average up

- **Reality**
  - With no uncertainty, the LOOP plus a view about the short-term interest rate give a complete characterization of the term structure
  - Tell me about the future path of short rates, and I'll tell you what long-term rates are
  - Tell me about current long-term rates, and I'll tell you about the future path of short-term rates

- **Uncertainty.**
  - Once we bring uncertainty in, the theory of the term structure gets much more subtle.
  - The first theory we'll discuss trivializes this complexity

    This is the smallest possible modification of our LOOP+certainty theory to account for uncertainty.

- **Expectations theory of term structure**

- Under certainty, we said:

  Any two ways of investing for M periods must pay the same

- In the expectations theory we say that any two ways of investing for M periods must be expected to (that is, on average) pay the same return

  And by 'expected' we mean the statistical sense of expectation.

- Simple version. Take our LOOP theory equation. Add uncertainty.

  Then, simply replace any unknown future-dated values with their expected value

- LOOP+certainty:

$$(1+i_{10,t})^{10} = (1+i_{3,t})^3(1+i_{7,t+3})^7$$

- Expectations theory+uncertainty makes one change:

$$(1+i_{10,t})^{10} = (1+i_{3,t})^3(1+i^e{}_{7,t+3})^7$$

  And as always the e means the expected value of the item

- **Similarly,**

  -

$$(1+i_{10,t})^{10} = (1+i_{1,t}) \times (1+i^e_{1,t+1}) \times \ldots (1+i^e_{1,t+9})$$

  or doing our same approximation as before:

$$i_{10,t} \approx \frac{1}{10}\left(i_{1,t} + \sum_{j=1}^{9} i^e{}_{1,t+j}\right)$$

  where now we have an e on all the 'future' 1-year rates.

- **Short hand for expectations theory of the term structure: Long-term interest rates (approx.) equal the average of expected future short-term rates.**

- **Expectations theory: implications**

  - If today's 10-year rate is above today's 1-year rate then, by the expectations theory, the market expects the 1-year rate to increase

    by the same reasoning as in the LOOP+certainty case.

- **Adding a realistic treatment of uncertainty**

- **Economic content of the expectations theory**

- The 'expectations theory' is essentially the theory asserting that on average risk doesn't matter

  Going from certainty to uncertainty, just put an e superscript on unknown stuff

- Taken literally, this assertion goes against everything we've learned.

  Folks care about risk...

- More carefully: folks will pay to be exposed to 'good' risk, and expect to be paid to bear 'bad' risk
- In principle, the expectations theory could be close to right if on average risk doesn't matter much.
- It turns out that risk does matter, so we need a richer theory.

  Let's talk this through

▶ **Risk in the term structure**

- Compare buying a 10-year bond to buying a 5-year and rolling the proceeds into the next 5 year bond.

  (assume there is no default risk)

- With the 10-year bond, the nominal return is fixed ...
- ... but the real return you'll receive is uncertain

  Inflation could end up being higher or lower than expected

- If inflation risk is my main concern, buying the current 5-year bond and rolling into the next one is better.
- If expected inflation has risen or fallen after 5 years, this will be incorporated in the yield I earn when I buy the second 5-year bond
- If I buy the 10-year, I am locked in to a certain nominal return for the whole period.
- On the other hand, suppose that inflation is highly predictable so we don't have to worry about that.
- But assume that the real return in the economy changes (say, it falls)
- In this case, when I roll over the first 5-year bond into the second one, I will recieve the new lower real rate of return

  If I had bought the 10-year, I would have locked in the higher real return for the full 10 years.

▶ **...** Which should the investor prefer?

[10-year, or 5-year rolled into 5-year]

- ... The answer is the same answer that a good economist gives to almost every question: it all

  depends

- **Term or risk premium?**
  - We've said that folks demand a return premium to face bad risk.
  - So in reality should which investment strategy (10-year bond or 5-year rolled into 5 year) should offer a premium *ex ante* ? It depends
  - As an empirical fact, most of the time, folks seem to demand a positive premium to hold the 10-year versus rolling over shorter bonds.
  - That is, on average when short-term interest rates are expected to be about constant, the yield curve slopes upward (at least a bit)

    Because of the yield premium on longer-term bonds

- **Liquidity premium**
  - We will follow the text in calling the risk compensation in the term structure a 'liquidity premium'
  - And so we augment the standard equation implied by expectations theory by adding a premium
  - liquidity premium theory

$$i_{10,t} = \frac{i_{1,t} + \sum_{j=1}^{9} i^e_{1,t+j}}{10} + \ell_{10,t}$$

    Where $\ell_{10,t}$ is the risk or liquidity premium the market pays to compensate for risk in the 10-year security.

  - In principle, ℓ could be either positive or negative
  - And in practice, it appears that usually in the market ℓ is positive, but sometimes it is negative

- **Aside:: The term 'liquidity' premium**
  - This use of the term 'liquidity' is one of the many examples where the somewhat foggy and often ill-defined notion of liquidity is invoked in finance.
  - This is very common, but you should never presume when you hear the word 'liquidity' that the speaker has any clear notion of what he/she means.

    Sometimes he/she will have a clear idea, often not.

- **Other theories**
  - The book lists segmented market theory

and in footnote: preferred habit theory.

- Just ignore these

▶ **Reality**

- In reality, there seems to be large and variable premia: that is, the expectations theory is does not hold in practice.

  And it occasionally changes sign (goes negative)

- Despite knowing this, you will often hear people (pundits, economists, policymakers, etc.) reason based on the expectations theory.
- Thus, if long-term rates are above short-term rates, folks regularly state that 'markets expect short-term rates to increase'

▶ **Macaulay**

- In the 1930s, Macaulay looked at such predictions and concluded

  'Now experience is more nearly the opposite.'

- When an upward sloping yield curve predicts rising short rates under the expectations theory, short rates subsequently tend to fall instead

▶ **Cite**

- The Macaulay quote and more is in,

  Do Long-Term Interest Rates Overreact to Short-Term Interest Rates? N. Gregory Mankiw, Lawrence H. Summers and Laurence Weiss Brookings Papers on Economic Activity, Vol. 1984, No. 1 (1984), pp. 223-247 go

  http://www.jstor.org/stable/2534279

▶ **Up until the crisis, at least**

- Up until the crisis, at least, experience remained as in the 1930s
- But (inexplicably) people still speak as if they take the expectations theory as a good predictor

▶ **Alan Blinder**

- Famous economist, former vice chairmain of Fed,

  Yet everyone—and here I mean analysts, market participants and central bankers alike—continues [despite the evidence] to "read" the market's expectations of future short rates from the yield curve, as if doing so made sense. I find it hard to explain why everyone is doing what everyone knows to be wrong.... (1997, p.16)

- Cite

  Blinder, Alan, Distinguished Lecture on Economics in Government: What Central Bankers could learn from Academics—and Vice Versa, Journal of Economic Perspectives, vol. 11, no. 2, Spring 1997.

▶ **Case study: Recent long-term rates in U.S.**

- From mid-November 2014 to mid-January 2015, the yield on 10 year U.S. government bonds fell from about 2.4 percent to 1.7 percent.
- Q:If long rates fall, what does the expectations theory say has happened to expected future short rates?
- A: Expected future short rates must have fallen
- But over this period the Fed had been communicating a shift up in the likelihood that short-term rates would soon rise from zero.
- This suggests a problem for the expectations theory:

  long rates down, expected future short rates up.

▶ **Case study: Two possibilities**

- 1. Folks don't believe the Fed

  Perhaps they think that the bad economic news we hear from abroad will soon come to the U.S. and that the Fed will not raise rates.
- 2. Premia in the 10-year rates are falling

  Say, risks abroad have risen, shifting out the demand for U.S. bonds and decreasing the premium demanded for holding them
- Note: the phenomenon that U.S. government bond rates fall when the world gets scarier in some way is called a 'flight to safety' effect.

▶ **Bottom line**

- Expectations theory has a large grain of truth.

  Expected future short rates must (in any reasonable theory) be reflected in current long rates
- Expectations theory is a good starting point:

  Long-term rates are the average of expected future short rates, plus a very tricky liquidity premium
- Bit remember that Liquidity premia are large and quite variable in practice.

▶ **Thus,**

- When long-term rates change, you can almost always tell a story driven by changing expectations of future short rates
- And you can also tell a different story driven by changing premia.
- You should get used to thinking up both kinds of story and forming a judgment about which is more plausible.

▶ **Finally some terminology**

- Financial markets folks often speak of forward interest rates.
- These are closely related to our expectations theory discussion.

▶ **Forward rates**

- If I know the current 10-year rate and the current 3 year rate, I can ask,

  Q: What 7-year rate 3 years from now would make these two investments pay off the same?

- A: This rate will be f in:

$$(1 + i_{10,t})^{10} = (1 + i_{3,t})^3 (1 + f)^7$$

- Solving, we get

$$1 + f = \left(\frac{(1 + i_{10,t})^{10}}{(1 + i_{3,t})^3} \right)^{1/7}$$

- f in the above equation is known as the 7-year forward rate, 3 years hence.

  Or the 3-year forward, 7 year rate.

- Market folks often talk of the 5-by-5

  5-year forward rate 5 years hence.

▶ **Generally,**

- The M-year forward rate S years in the future is

$$1 + f = \left(\frac{(1 + i_{M+S,t})^{M+S}}{(1 + i_{S,t})^S} \right)^{1/M}$$

▶ **Another way of describing the expectations theory** Expectations theory of term structure

says that forward rates *are* the market's expecation of future spot interest rates

▶ **Note: You can look at forward rates on Bloomberg**

▶ **Forward rates on Bloomberg**

- To see U.S. forward rates at any point use the command:

  FWCM ¡go¿