The yield curve, maturity transformation and interest rate risk
266: Financial Markets and Institutions

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http://e105.org/e266

March 16, 2017

▶ Liquidity premia

- When we studied the term structure we settled on a theory that related long-term rates to short-term rates in a simple way.
- In words: long-term rates are the sum of two components
  - The average of expected future short-term interest rates, and
  - A premium that we called a liquidity premium.

▶ In math

- In a formula, we said:

$$i_{10,t} \approx \frac{i_{1,t} + \sum_{j=1}^{9} i_{1,t+j}}{10} + \ell_{10,t}$$

where $\ell_{10,t}$ is the term premium for 10-year bonds prevailing at time t.

▶ Liquidity premium

- We said that the premium was usually positive: folks demand a higher interest rate to hold the 10-year bond than to roll over the sequence of short-term bonds.
- Why? Well the answer is that we don't really know the full story
- But: One important part of the story is that short-term bonds are more liquid than long-term bonds.

  They can more easily/reliably be converted into something widely usable in transactions at a predictable price.

▶ Liquidity premium, again

- Take two assets that are the same in all respects except that one can more easily be converted into cash and is thereby more liquid.
- People will be willing to hold the more liquid one at a lower rate of return.

▶ **Aside:: Negative interest rates and the liquidity of cash**

- Why are people in many countries in the world holding assets that pay negative interest rates?
- This might seem particularly puzzling given that folks could instead just hold cash and make 0%.

  At least they wouldn't be losing money

- One reason is that some things are more liquid than cash.
- A short-term government security is more liquid than cash
  - It has very stable value (similar to cash)
  - Can be stored securely and moved in large values pretty cheaply

    Electronic entries are easier to move around than dollar bills

▶ **Aside:: Liquidity in modern finance**

- Much of modern finance until recently viewed the liquidity premium as a nuisance and ignored it in many contexts

  (e.g., remember the Blinder quote about everybody ignoring the liquidity premium in reasoning from the term structure)

- Especially with the advent of negative rates, it has become harder to ignore liquidity premia.

▶ **Liquidity and maturity**

- Last time we introduced the idea that when interest rates change, this leads to a larger change in value for longer-term bonds than for shorter-term bonds.
- In a world of changing interest rates, then, the price of longer-term bonds tends to be much more variable or volatile than that of shorter-term bonds.
- But this implies that longer-term assets are less liquid.
- Part of liquidity is having a good sense of the value of the asset.

▶ **Volatility and liquidity**

- You know you have $1500 rent coming due next week.
- You have $1500, but you are deciding what asset to hold that $1500 in for the next week.

- You could put it in your checking account, or buy a 10-year government bond, or buy stock in, say, a Pharmaceutical company that is rumored to be working on a hot new drug.
- You are pretty sure you'll be able to make the rent payment with the first option,
- The 10-year bond could rise or fall in a value a good deal over the next week, so its not so reliable.
- That Pharma company stock? You'd be nuts to put your rent payment there for a week.

  even if is a good investment over the longer term.

▶ **Thus,**

- Maturity and liquidity are closely linked: longer maturity, less liquid.

▶ **Banks, liquidity, and maturity transformation**

- Banks provide liquidity to the rest of the economy
- Remember that the bank's liabilities are everybody else's liquid assets.
- And those liquid assets the bank offers to others are very short-term: you can get your funds pretty much any time.

▶ **Bank loans**

- Bank assets tend to be much longer in maturity than their liabilities.
- Borrowers would not take out a loan if the bank said 'Oh, and I have a right to demand repayment at a moment's notice.'
- Loans tend to be of much longer maturity than the deposits.

▶ **Aside:: Callable loans**

- Callable loans are loans where the bank can demand repayment prior to the agreed term of the loan
- But even in this case, the borrower usually has something like 30 days to re-pay.

▶ **Banks**

- We say the banks perform *maturity transformation*
- Short-term money comes in the door in the form of deposits; longer-term loans go out the door.

  Banks thereby have a *maturity mismatch* in their assets and liabilities

- Since longer-term instruments are less liquid, we can also say that banks perform liquidity transformation they fund illiquid loans using liquid deposits.

- **Banks are magic?**
  - Q: Is this some sort of black magic?

    Do banks somehow have a magical liquidity machine in the back room?

  - A: Nope.

    that's why we have bank runs, unless there is a government fix: if the liquid deposits are demanded early, the bank cannot demand repayment of the loans and (unless someone steps in) the bank goes under.

- **Managing risks due to maturity gap**
  - The risks arising from the change in asset values when interest rates change is called *interest rate risk*
  - A primary element of risk managment at a bank is the managing interest rate risk arising from maturity transformation

    (later, we'll talk about a several other categories of risk)

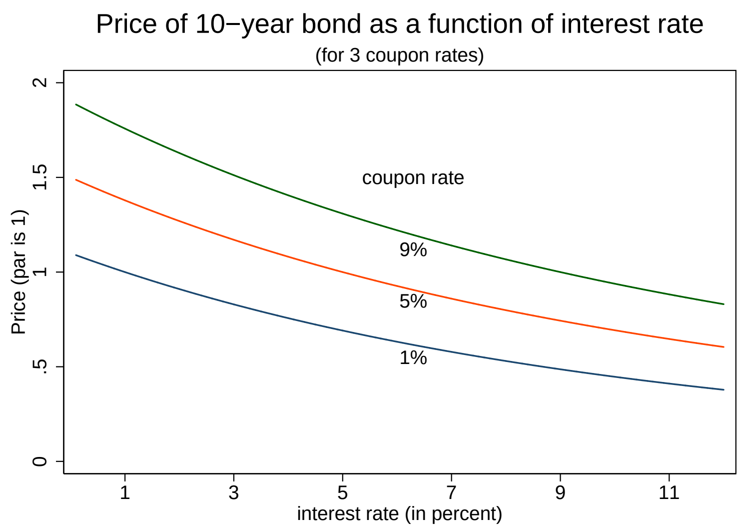
- **Example from last time**
  - We introduced this last time with our simple bank that offered 1-month deposits and 7-year loans.
  - Thus, everything the bank owed was coming due in 1 month; everything the bank was owed was coming due in 7 years.
  - In reality, banks have a big collection of liabilities and assets with payments coming in and going out at lots of horizons between a day and 30 years or more in the future.
  - We know that the present value of longer-term payments falls by more than shorter-term payments if interest rates at all horizons rise
  - But to manage the risk invovled we need to know more precisely by how much the values change.
  - And this takes us back toward the rocket science (mathy) side of finance.

- **Managing interest rate risk when you have a bunch of assets and liabilities with payments coming at different maturities.**

- **Interest rate risk from a bond**
  - Let's illustrate the ideas in the context of a single coupon bond.

- **Coupon bond**
  - Remember a plain vanilla, $h-$year coupon bond pays c each year for h years and in the final year pays $c + F$

    where c is the coupon payment and F is the face, or par, or principal value.
  - We are interested in how the value changes when interest rates change



- **Note**
  - At higher coupon rates, the price is more sensitive to interest rates
  - At lower interest rates, the bond price is more sensitive to any given change in interest rates
  - Let's dip into the math behind this picture
- **Present value**
  - The present value of this flow of payments is given by,

$$V_0 = \sum_{j=1}^{h} \frac{c}{(1+i_0)^j} + \frac{F}{(1+i_0)^h}$$

    where i_0 is the constant interest rate prevailing initially.
  - If this constant interest rate rises to i_1, we have

$$V_1 = \sum_{j=1}^{h} \frac{c}{(1+i_1)^j} + \frac{F}{(1+i_1)^h}$$

  - And the change in value, $V_1 - V_0$ is a complicated function of h powers of i_0 and i_1.

Being a high order polynomial, this formula is a bit of a pain to work with.

- **Approximation**
  - As we'll explore today, using an approximation akin to $\ln(1+z) \approx z$ the percent change in value, or rate of capital gain (RCG) can be approximated fairly well by a much simpler formula:

$$RCG \approx -DUR \times \frac{i_1 - i_0}{(1+i_0)}$$

    where $RCG = (V_1 - V_0)/V_0$ and DUR is a number summarizing the bond (more on this next slide)
  - (Note: I say RCG for 'rate of capital gain;' the book says $\%\Delta P$ for the same thing.)
- ***DUR* is duration**
  - Informal: Duration is a single number summarizing the 'effective maturity' of a collection of payments coming at different times in the future.
  - More formal: We'll give a mathematical (Taylor series) account below
- **Informal account of duration.**
  - You have a bunch of payments coming at a bunch of different times in the future.
  - But you want to treat them as if they were (approximately) a single payment coming at a single maturity.
- **Duration**
  - Q: What maturity should you choose?
  - Whenever you think about good approximations, you need ask 'good for what purpose?'
  - If your main purpose is to summarize interest rate risk, then duration, DUR is probably the best single measure of effective maturity.
- **And how do financial institutions use *DUR*?**
  - Suppose a bank that wants to manage interest rate risk—that is, the risk of capital gain or loss it faces when interest rates change.
  - It gathers up all the payments coming in on its assets and computes their duration, DUR_a.
  - Then it gathers up all the payments that will go out on the liabilities and computes the duration: DUR_l.
  - And now we can compute the (approximate) rate of capital gain on both assets and liabities using the formula for RCG given above.

- **Back to our initial bank example**

- **Balance sheet**

Our firm

Assets		Liabilities	
loans	1000	deposits	1000
other	30		
		net worth	30
total	1030	total:	1030

- **But now...**
  - Assume that the deposits overall have a duration of 1 month.
  - And loans overall that have a duration of 7 years.
  - What will be the approximate change in net worth if interest rates go from $i_0 = 0.03$ to $i_1 = 0.035$?

- **Remember**
  - Well, we answered that last time: net worth falls by about $33.
  - We could also get that value using the formula:

$$RCG = \frac{(NW_1 - NW_0)}{A} \approx -DUR_{gap} \times \frac{(i_1 - i_0)}{(1 + i_0)}$$

    where

$$DUR_{gap} = DUR_a - (A/L)DUR_l$$

    and DUR_a and DUR_l are the durations of assets and liabilities, respectively.
  - In the example, $DUR_{gap} = 7 - (1030/1000)(1/12) \approx 6.91$, and using i_0 and i_1 from above

$$RCG \approx 0.033$$

    and 3.3% loss on $1,000 gives us the same number as before $-\$33$

- **To see this more slowly...**
  - Read the duration gap analysis part of chapter 23.
  - Of course, in our case with only 2 maturities to deal with, it is just as easy do calculate the exact number as this approximate one
  - The example in the book is richer—the bank has liabilities and assets paying off at lots of different points in time.

- **Reality**
  - In reality, this kind of duration analysis plays an important role in financial markets
  - As a rough guide to interest rate risk management the bank can compute the duration gap
- **Managing interest rate risk**
  - If the duration gap is too large, what can the bank do?

    As we'll see, a main thing banks do is to reduce the effective maturity of assets various ways
  - We'll do more detail later.
- **For now:**
  - Banks engage in maturity and liquidity transformation
  - This means that they face and must manage interest rate risk.
  - Duration is an important concept for banks and all financial institutions.
- **Let's take a detour into mathy stuff (not on the exam)**
- **Math and reality**
  - In lots of arenas of life, events, outcomes, behavior can be described pretty well, or even extremely well, by equations.
  - But often those equations are really complicated and impractical to work with
  - Often the equations are so complicated we can't even write them down explicitly or, if we can write them down, we can't solve them
  - This does NOT make the equations useless for practical purposes
  - In particular, often there are formal approaches to approximating the formulae
  - And the approximations are easy (or much easier) to work with
  - And the approximations are sufficiently good—they capture enough of the essence of the 'true' equation—that they are widely used.
- **General Relativity**
  - General relativity is a very deep application of this idea
  - The equations can't be solved
  - We have approximated them various ways, and much of this area of Physics has been about solutions to approximations and what they mean and whether they describe reality.
  - Bernard Shutz (Albert Einstein Institute) put it this way

> Relativists seem to take a perverse pride in the fact that Einstein's equations are hard to solve.... The difficulty of solving equations means that in all branches of physics progress can normally be made only by using idealizations and approximations.

► **Cite**

- Shutz, B.F.,

  The use of Perturbation and Approximation Methods in General Relativity. In: Relativistic astrophysics and cosmology: Proceedings of the GIFT International Seminar on Theoretical, (Eds.) Fustero, Xavier; Verdaguer, Enric; Singapore: World Scientific 35-98 (1984) go

  http://www.aei.mpg.de/ schutz/papers/schutz%5F60305.pdf

► **For those with a bit of calculus**

- There are many formal forms of mathematical approximation.
- Many of you may have learned the famous Taylor series approximation in calculus 1.
- You can go learn about, or review, Taylor series and other approximations at many places on the web.

  But you won't need to know it for this class.

► **Taylor-type approximations in finance**

- Suppose the value of an asset can be written as a function of the prevailing interest rate:

$$PV_0 = f(i_0)$$

  and after an interest rate change we have

$$PV_1 = f(i_1)$$

- To protect ourselves from the risk of this change in rates, we need a convenient way to calculate changes in value like $f(i_1) - f(i_0)$.
- The Taylor series provides a sequence of increasingly accurate approximations to the change in value:

$$\begin{aligned} f(i_1) - f(i_0) &\approx (i_1 - i_0)f_1(i_0) \\ &\approx (i_1 - i_0)f_1(i_0) + \frac{1}{2!}(i_1 - i_0)^2 f_2(i_0) \end{aligned}$$

  and so forth, where f_j is the j^{th} partial derivative of f with respect to its sole argument.

- For very small changes in interest rates, the first approximation is quite accurate

  this is called a first-order approximation

- The second-order approximation tends to be more accurate for a given small range of changes in i,

▶ **Wall Street words**

- Our duration formula is (after switching to rate of change terms) a first order approximation
- In the context of financial options, taking account of risk as captured in the the first term of the approximation is called 'delta hedging' and taking account of the second is called 'gamma' hedging.

  For a taste, see the Wikipedia entry go

  https://en.wikipedia.org/wiki/Delta%5Fneutral#Theory

- This little diversion was to give you a sense of how the mathy side of finance works

  won't be on the exam.

▶ **Finally: the formula for duration**

- Take a generic cash flow, $s_1, \ldots, s_M$

  the amount s_j comes in j years.

- Call the present value of each of the payments, $PV_1, \ldots, PV_M$:

$$PV_j = s_j/(1+i)^j$$

- The present value of the full stream is the sum of these

$$PV = \sum_{j=1}^{M} PV_j$$

- Define w_j, the share of all of the present value that comes in the j^{th} year:

$$w_j = \frac{PV_j}{PV}$$

- And since, the w_js are shares, they sum to 1,

$$\sum w_j = w_1 + \ldots + w_M = 1$$

- Duration of the cash flow is defined as

$$DUR = 1w_1 + 2w_2 + \dots Mw_M$$

or

$$DUR = \sum_{j=1}^{M} j \times w_j$$

where w_j is the share of the pres. val. coming in the payment j years in future

▶ **Duration in words**

- Duration is a weighted average of all the times to payment, where the weights are the share of the total present value contributed by the payment at that point in time.

▶ **Aside:: terminology**

- The definition I have given is for 'Macaulay duration'

You can google 'Macaulay' vs. 'modified' duration

▶ **Aside:: Calculus**

- For those with calculus, both Macaulay and modified duration can be understood by playing with $\partial \ln(PV)/\partial i$
- Or go to the bond duration Wikipedia page

go

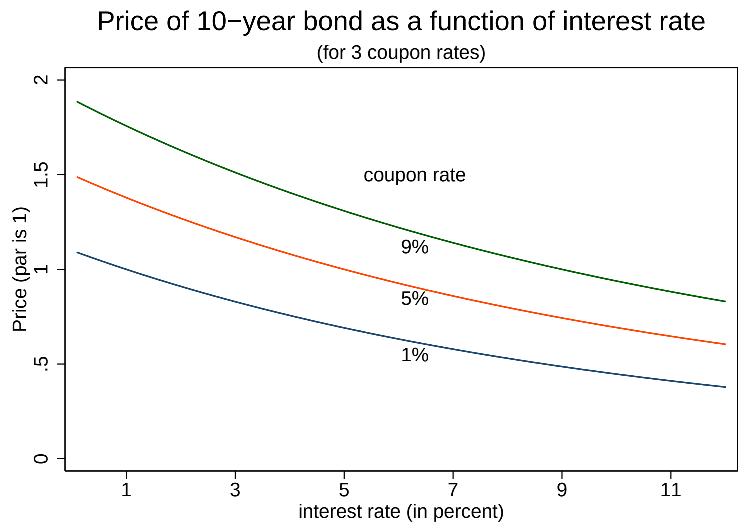
https://en.wikipedia.org/wiki/Bond%5Fduration#Macaulay%5Fduration

▶ **Important factoid**

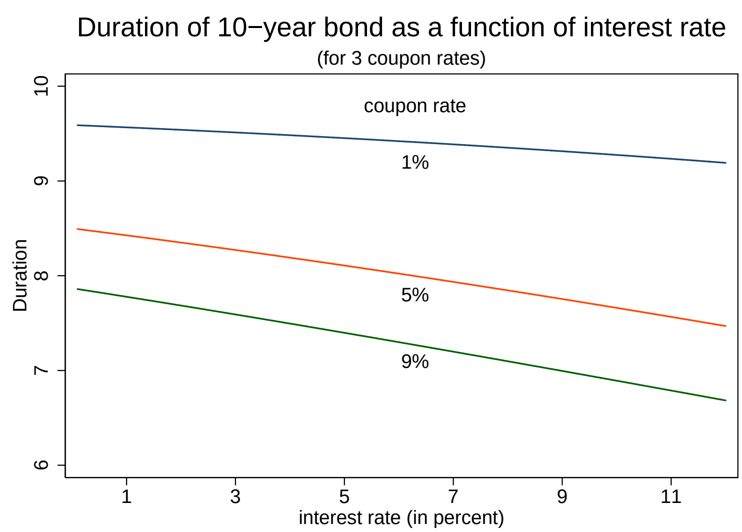
- For a single payment (say, an IOU or zero coupon bond or Treasury bill), what is the duration?

Duration equals maturity for a single payment. You can verify this with the formula

▶ **Back to the bond price picture**



- **The analogous duraton picture**



- **Note:**
  - Higher coupon rate, all else equal means lower duration

    You get more of your value back earlier (in the coupon payments)

  - Higher interest rate implies lower duration

    Those later payments contribute less to value when they are discounted at a higher interest rate.

  - Finally, the change in duratinon as interest rates change is nearly linear (but the lines have a bit of curve)

- **Next time:**
  - Other aspects of risk management at financial institutions.