Name: _____

Problem set 1
266: Fi. Markets and Institutions
Spring 2011
Jon Faust

**Directions.** You are to do this problem set alone. If you would benefit from conferring on how to do the part that is in Excel, this is ok, but complete the work yourself.

**Due Date/time.** Your work is due by the beginning of class (12:00 PM) March 10. You can hand the work in to me at the beginning of class. If you put the work under my office door or in my mailbox, it must be in before I leave for lecture at about 11:55pm. Only hardcopy submissions allowed.

**Questions.** If you have questions, email me or the TAs or raise them in class, or come to office hours.

**Grading.** All parts have equal value.

## 1 Basics

1.1 I own a stock for 5 months. I bought for $30.23 and sold for $37.52. What is my annualized rate of capital gain in percent?

**Answer:** Recall that capital gains are the returns associated with a rise in the price of an asset (as opposed to cash payments, such as coupon or dividend payments). This just an application of the usual formula:

$$1 + i = \left(\frac{\text{proceeds}}{\text{costs}}\right)^{\frac{1}{\text{holding period}}}.$$

The proceeds are simply the price at the time of sale and costs are the original purchase price. The holding period is $\frac{5}{12}$ of a year. So,

$$
\begin{aligned}
1 + i &= \left(\frac{37.52}{30.23}\right)^{\frac{1}{\frac{5}{12}}} \\
1 + i &= \left(\frac{37.52}{30.23}\right)^{\frac{12}{5}} \\
i &= \left(\frac{37.52}{30.23}\right)^{\frac{12}{5}} - 1 \\
i &\approx .6795
\end{aligned}
$$

So the annualized rate of capital gain, in percent, is 67.95%.

1.2 The interest rate on a risk free zero coupon bond with maturity 3 years is 2.21 percent. The interest rate on a risk free zero coupon bond with maturity 7 years is 2.63 percent. What is the implied forward rate for a 4-year bond 3 years from today?

**Answer:** This is just an application of the formula discussed in class:

$$
\begin{aligned}
(1 + i_{3,t})^3 (1 + i_{4,t+3})^4 &= (1 + i_{7,t})^7 \\
(1.0221)^3 (1 + i_{4,t+3})^4 &= (1.0263)^7 \\
(1 + i_{4,t+3})^4 &= \frac{(1.0263)^7}{(1.0221)^3} \\
1 + i_{4,t+3} &= \left(\frac{(1.0263)^7}{(1.0221)^3}\right)^{\frac{1}{4}} \\
i_{4,t+3} &= \left(\frac{(1.0263)^7}{(1.0221)^3}\right)^{\frac{1}{4}} - 1 \\
i_{4,t+3} &\approx .0294.
\end{aligned}
$$

The implied four year interest rate in three years time is 2.94%. This is an illustration of the conclusion from expectations theory that if long-term interest rates are higher than short-term interest rates, then short-term interest rates must be expected to rise. This follows from the fact that, in expectations theory, long rates are an average of current and future short rates. So if the long rate (the average) is larger than the current short

rate (the first term in the average), then the terms that come later in the average (future short rates) must be above the average (long rates). Of course, as we have a discussed in lecture, this conclusion has proven to be rather unrealistic, a reflection of fundamental problems with expectations theory. We can also think of the formula we used above in intuitive terms. In our simplified theory, we know that the return from holding a seven year bond must be the same as the return from holding a three year bond followed by a four year bond. If you aren't convinced of this fact, use the usual "what would happen if it weren't true …" argument. If, for example, the return from holding a sequence of a three- and then a four-year bond was higher than the return to holding a seven year bond, demand would rise for the three- and four-year bonds (raising their price and thus lowering their yields) and fall for the seven year bond (lowering the price and raising the yield). So long as the two returns aren't equal, this story will tend to push them to become equal. This can also be thought of (loosely, at least) as an example of the law of one price: The pair of bonds sequentially and the single bond are basically identical assets so they should be priced the same.

1.3 Today's one-year interest rate is 1.03 percent and the 5-year rate is 2.28 percent. According to the expectations theory of the term structure, short-term interest rates are expected to fall, rise, or stay the same?

**Answer:** Short term interest rates are expected to rise. This follows from the fact that, in expectations theory, long rates are an average of current and future short rates. So if the long rate (the average) is larger than the current short rate (the first term in the average), then the terms that come later in the average (future short rates) must be above the average (long rates). Of course, as we have a discussed in lecture, this conclusion has proven to be rather unrealistic, a reflection of fundamental problems with expectations theory.

1.4 A 10-year bond pays a coupon rate of 3 percent and pays annual coupon payments. Its yield to maturity is 5 percent. Is the bond

selling below par or above par?

> **Answer:** This bond is selling *below* par. A bond is selling *at* par if the coupon rate is equal to the yield to maturity. In this case, the yield to maturity is above the coupon rate, so the price must be below the par value (since prices and yields move inversely).

2  Use a spreadsheet program or financial calculator to do the following.

2.1  I just signed a 30-year, fixed-rate mortgage (360 equal monthly payments). My monthly mortgage payment is \$1,650. My mortgage interest rate is 4.56 percent. How much did I borrow?

Hint: What we want here is the present value of the payments when discounted at the mortgage interest rate. Assume that the first payment comes 1-month after signing.

> **Answer:** The answer to this question depends upon whether you used simple or compound interest. If you used simple interest, the answer is \$323,366.71; if you used compound interest, the answer is \$326,886.73. I will explain how to obtain this answer in Microsoft Office Excel 2007. Other programs or calculators may vary somewhat, but the basic points are the same.
>
> In an Excel spreadsheet, choose an empty cell to place the formula. Go to the "Formulas" tab, open the "Financial" drop-down menu, and choose the function "PV." A window pops up that has five fields in it. The first three (whose names are in bold face) are required, while the last two are optional.
>
> > • The first field ("Rate") is the *per-period* interest rate. Note that this function allows only one payment per period, so you have to make the annual interest rate into a monthly one. You can use simple interest, $4.56/12 = .38$, or compound interest,$(1.0456)^{(}1/12) = .3723$. Although it's not clear whether the interest rate should be in percent or decimal notation, it needs to be in decimal notation. So you should enter .0038 or .003723 in the first field. In using all financial software/calculators, be careful about the period of the interest rate (annual, quarterly, monthly, ...) and what form the interest rate needs to be in.

- The second field ("Nper") is the number of periods. Since a period is month and this a 30-year mortgage, there are 360 periods.
- The third field ("Pmt") is the payment per period. This is $-1650$. The payment is negative because it is a payment made *by* us, not *to* us. If you leave it positive, you get the correct number but the opposite sign (from the perspective of the borrower, a mortgage has negative present value, since it's going to cost him/her money).
- The fourth field ("Fv") is the future value of the asset. That is, how much it's worth at the end. In this case, it will be worth $0 in 30 years (since it will be paid off). Since Excel interprets this field being left blank as a "0," we can leave it blank.
- The fifth field ("Type") specifies whether the payment is made at the beginning or end of the period. In our case, it's the end (since the question says that the first payment comes one month after signing). Excel interprets this field being left blank as a "0," which corresponds to the payment coming at the end, so we can leave the field blank (to have payments come at the beginning, enter a "1").

Press enter, and you have the answer. Note that you could also have simply typed "=PV(.0038,360,-1650)" or "=PV(.0038,360,-1650,0,0)" in the cell and hit enter. Note also that putting an equal sign at the beginning of any cell tells Excel that the text which follows it is a formula (as opposed to ordinary text, numbers, etc.).

2.2 Just after signing the mortgage, interest rates fall to 4.30 percent. Now what is the present value of what I owe?

**Answer:** The present value is now $333,420.09 for simple interest and $336,687.74 for compound interest. This answer is obtained by changing the interest rate from 4.56% to 4.30% (or .35833 for simple interest or .3515 for compound interest) in the Excel function used in part 1. The change in the interest rate just changes the rate at which future payments are discounted in the present value equation we have talked about in class. This is yet

another demonstration of the fact that prices and yields move inversely.

2.3 What is the duration of a 30-year coupon bond (annual coupons), paying a coupon rate of 6 percent, and selling at par?

**Answer:** The bond has a Macauley duration of 14.59. To obtain this result in Excel 2007, go to the "Formulas" tab, open the "Financial" drop-down menu, and choose the function named "DURATION." A window pops up that has five required fields in it.

- The first field, "Settlement," is the date at which the security is purchased. For our purposes just pick a date (like 1/1/00). The format that you have to enter the date in is not immediately obvious, however. You have to enter the date by typing "DATE(yyyy,mm,dd)," so that 1/1/2000 is entered as "DATE(2000,1,1)."

- The second field, "Maturity," is the date, in the same format as above, at which the bond matures. Since we want a 30-year bond, pick the date 30 years after the settlement date (1/1/2030 in my example).

- The third field, "Coupon," is the bond's annual coupon rate. This is 6%, but it should be entered as 0.06. As usual, with other programs and calculators, be careful about what form interest rates are in.

- The fourth field, "Yld," is the annual yield on the bond. Although the question doesn't explicitly state the bond's yield, it does say that the bond is selling at par. As we know from question 1.4, a bond is selling at par if the coupon rate equals the yield to maturity (and vice versa). Thus, the yield is 6%, or 0.06 in decimal form.

- Finally, the fifth field, "Frequency," is the number of coupon payments per year: 1.

Hit enter and you are done. Note that you could also have simply typed "=DURATION(DATE(2000,1,1),DATE(2030,1,1),0.06,0.06,1)" into the cell and hit enter.

Note also that there is another function, "MDURATION," which you may be tempted to use. Excel says that DURATION returns the "annual duration," while MDURATION returns the "Macauley modified duration." This

may be confusing, since we used the names "Macauley" and "modified" for the two measures of duration. However, from the lecture notes, we know that Macauley duration (as defined in class) is the modified duration over $1 + i$. If you try it, you'll see that DURATION returns a number that is equal to MDURATION over $1+i$. So, DURATION corresponds to Macauley duration (as we've defined it) and MDURATION corresponds to modified duration (as we've defined it). Modified duration in this case is 13.76.

3 Stocks and bonds. Use a spreadsheet ps12011data.xls you can download in conjunction with the problem set for this problem. The spreadsheet has some inflation, interest rate, and stock returun data for 1962 to 2010.

Note: The nominal stock portfolio data are from Ken French's website. This is a fabulous source of portfolio data for study.
url: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Inflation is for the CPI from the St. Louis Fed Fred database. The 1-year Treasury rate is the 1-year constant maturity rate also from Fred. This is not strictly a zero coupon rate, but we will treat it as if it were.
Fred url: http://research.stlouisfed.org/fred2/

The spreadsheet has two pages; One page is for nominal returns and one for real returs. See the tabs labelled 'nom' and 'real' at the bottom.

The nominal return page is complete and can be used as a model for finishing the real return page.

Both pages have columns for 17 sector-based portfolios of stocks and also have the rate on a 1-year U.S. Treasury security.

A 'Market' portfolio return is computed based on a simple average of the returns in the 17 sectors. This is the approximate return you would get if you held a portfolio equal-weighted in the 17 sectors.

For more detail on the data, go to the websites listed above.

**Note: All of the following concern the real returns**

3.1 Using the 'nom' sheet as a guide, complete the 'real' sheet. That is: Add the market return data and then below the time series

data for the returns, fill in the rows for the mean, standard deviation, and for the 17 portfolios, the correlation with the market portfolio.

**Answer:** In the spreadsheet on the course website, the portions of the 'real' sheet that were added to complete this question are highlighted in blue. The spreadsheet was completed using the following steps:

- To obtain the market portfolio return, take a simple average, using the command "AVERAGE" of all 17 industries within the same row (or year). That is, any cell in the market portfolio column is an average of the 17 cells to the right of that cell.

- For the industry mean returns, use the AVERAGE function again, but this time you average the column (or industry) that is above the cell, over all of the years where we have data.

- For industry standard deviation, use the STDEV function for the column over all the years where we have data.

- For correlation, use the CORREL function. Unlike AVERAGE and STDEV, which only have one argument (ie, you only choose one block of cells to use the function on), CORREL has two. Since you want to see the correlation between the market and any one industry, one of the arguments has to be the market portfolio column, while the other is the particular industry's column.

3.2 How much higher was the mean return on the market portfolio than the mean return on the 1-year Treasury over this sample?

**Answer:** The mean real return to the market was 6.31% higher than the mean real return to 1-year Treasurys. This is obtained by taking the difference between the mean market return of 8.03% (from cell E57) and the mean Treasury return of 1.72% (from cell D57).

3.3 How much higher was the standard deviation of the return on the market portfolio than the standard deviation of the return on the 1-year Treasury over this sample?

**Answer:** The standard deviation of real returns to the

market was 15.3 larger than the standard deviation of the real returns to 1-year Treasurys. This is obtained by taking the difference between the market return standard deviation (18.01, cell E58) and the Treasury return standard deviation (2.71, cell D58). Note that this finding along with the results in the previous question illustrate the risk-return trade-off: one way to make higher returns is to take more systematic (market) risk. Although the returns to the market are considerably higher, they are also much more variable (ie, risky). The Treasury returns are lower, but much less variable.

3.4 What is the range of correlation of the returns between the individual portfolios and the market portfolio.

**Answer:** The correlations of industries with the market range from a .62 to .89. In plain English, every sector tends to do well when the market does well and tends to do poorly when the market does, but they all have some individual variation and none track the market perfectly. The cells colored yellow in the spreadsheet correspond to this question and serve as an example of the MIN and MAX functions. Note that there is no function which, by itself, displays the range of a group of cells.

3.5 Plot the return on the finance sector portfolio (finan), the market portfolio, and the 1 year treasury. Attach a hardcopy of the figure to your answers.

**Answer:**

Returns to the market, financial sector, and Treasurys
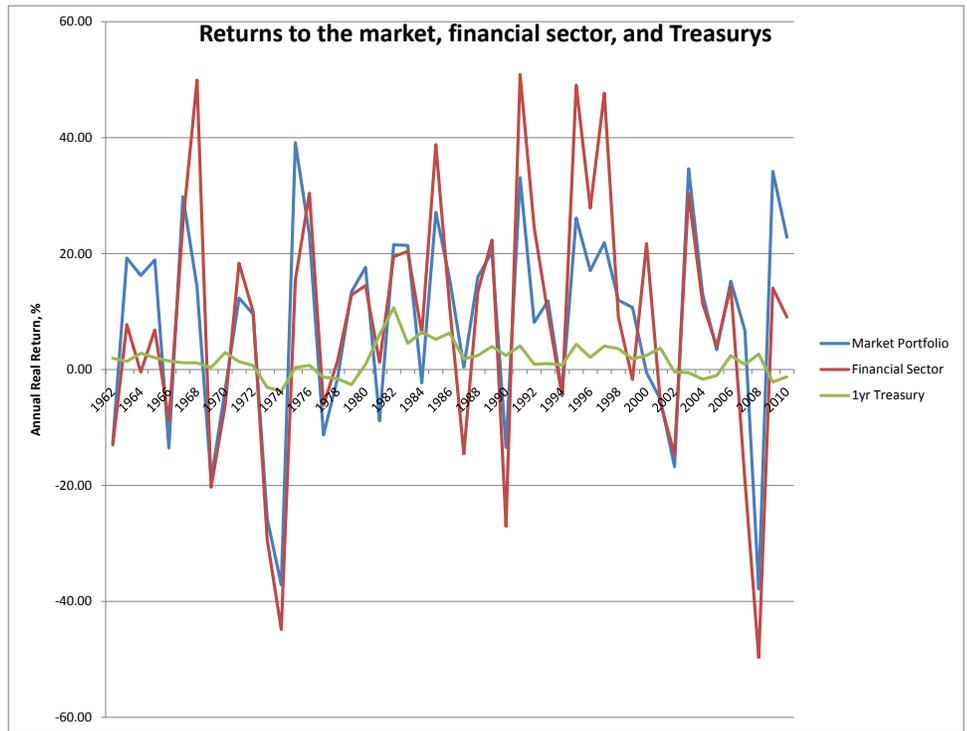
3.6 In the worst year in the sample, what share of your value would you have lost if you were invested in the financial sector? And what share could you have gained at best in the finnace sector?

**Answer:** The cells colored green correspond to this question. In the best year, 1991, you would have gained 50.9%. In the worst year, 2008, you would have lost 49.69%. To find these numbers, use the MIN and MAX functions, with the financial sector column as the function's argument.