Name: ______________________

Problem set 1: answers
266: Fi. Markets and Institutions
Spring 2015
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Directions. You are to do this problem set alone. If you would benefit from conferring on how to do the part that is in Excel, this is ok, but complete the work yourself.

Due Date/time. Your work is due by the beginning of class February 17. You can hand the work in to me at the beginning of class or bring it to my office before I leave for class at about 10:20am. Only hardcopy submissions allowed.

Questions. If you have questions, email me or the TAs or raise them in class, or come to office hours.

Grading. All parts have equal value (5 points each).

1 Percent changes

1.1 I invest \$500 today and get back \$550 in 3 years. In percentage terms, how much has my value increased?

Answer/comment

$$\%\text{increase} = \frac{550 - 500}{500} = 10\%$$

1.2 At what annualized rate did my value in the previous problem increase?

Answer/comment
Suppose the annualized rate is i.

$$\begin{aligned} 500 \times (1+i)^3 &= 550 \\ (1+i)^3 &= 1.1 \\ 1+i &= 1.0323 \\ i &= .0323. \end{aligned}$$

1.3 Someone offers me \$5 to come 18 months from now. If the relevant interest rate is 7.3 percent, what is the present value of this gift?

Answer/comment
18 months is 1.5 years.

$$\begin{aligned} PV &= \frac{5}{(1+\frac{7.3}{100})^{1.5}} \\ &= 4.50 \end{aligned}$$

1.4 An investment will pay \$7 in 1 year and and \$8 in two years. The current price of this investment is \$14.30. What is the internal rate of return?

Answer/comment
Suppose the internal rate of return is i. Then

$$14.30 = \frac{7}{1+i} + \frac{8}{(1+i)^2}$$

We can solve for i using a calculator. Or if we notice that this is a quadratic equation

$$14.30(1+i)^2 - 7(1+i) - 8 = 0$$

The analytic solution for $1+i$ is

$$\begin{aligned} 1+i &= \frac{-(-7) + \sqrt{(-7)^2 - 4 \times (14.30 \times (-8))}}{2 \times 14.30} \\ &= 1.0317 \\ i &= 3.17\% \end{aligned}$$

1.5 I bought a house at the beginning of 2007 for $200,000 and sold it at the end of 2007 for $150,000. What was my capital loss? What was my rate of capital loss over the year?

Answer/comment

$$\text{capital loss} = \$150,000 - \$200,000 = -\$50,000$$

$$\text{rate of loss} = \frac{\$50,000}{\$200,000} = 25\%$$

2 Probability and Statistics. Table 1 presents the prices of 4 assets (A, B, C, and D) in year $t+1$ under 3 different outcomes. The probabilities of each outcome are included. Note that none of the four assets makes any coupon or dividend payments, all prices are in \$, and t is measured in years.

Table 1

outcome	prob.	price at t+1			
		A	B	C	D
1	.3	5	18	16	18
2	.5	5	22	14	12
3	.2	5	26	12	23

2.1 Suppose that the price of asset A today (in period t) is \$4.854 . What is the expected return (in annual percentage terms) from buying A today and selling it in period $t+1$?

Answer/comment

This is just an application of the typical present value formula, once we know the expected payoff to the asset A. The expected value (that is, the mathematical expectation) for the payoff to selling asset A at time $t+1$ is just the mean payoff:

$$E[P_{t+1}^A] = .3(\$5) + .5(\$5) + .2(\$5) = \$5.$$

To find the expected rate of return, we now just plug the expected payoff into the present value formula:

$$\begin{aligned} 1 + i^e &= \frac{E[P_{t+1}^A]}{P_t} \\ 1 + i^e &= \frac{\$5}{\$4.854} \\ i^e &= \frac{\$5}{\$4.854} - 1 \\ i^e &= .0301. \end{aligned}$$

Thus, the annualized expected rate of return is 3.01%.

2.2 Given your answer to the above question, what should be the price of a portfolio consisting of one unit of asset B and two units of asset C? What, then, is the expected return to holding this portfolio?

Answer/comment

The trick here is noticing that the $B+2C$ portfolio is worth \$50 in all three outcomes for period $t+1$. That is, in each case, the sum of the prices of assets B and 2C at $t+1$ is \$50. Thus, the portfolio $B+2C$ has exactly the same payoff structure as a portfolio composed of 10 assets A. Since these two portfolios are identical, the law of one price says that they should sell for the same price. Thus, the price today (at time t) of the $B+2C$ portfolio is just

$$10 \times \$4.854 = \$48.54.$$

To find the expected return, we now just repeat the process from part 2.1, noting that $E[P_{t+1}^{B+2C}] = \$50$. That is,

$$\begin{aligned} 1+i^e &= \frac{E[P_{t+1}^{B+2C}]}{P_t} \\ 1+i^e &= \frac{\$50}{\$48.54} \\ i^e &= \frac{\$50}{\$48.54} - 1 \\ i^e &= .0301. \end{aligned}$$

Thus, the annualized expected rate of return is 3.01%. Of course, the rates of return on two identical assets must be the same, so this makes sense. Note that it is not the fact that both assets are risk free that is crucial in this problem, it is the fact that the portfolio is *identical* to asset A (that is, 10 units of asset A are the same as 1 unit of the portfolio) that is crucial.

2.3 What is the sign of the covariance of the prices of assets B and C? What is the sign of the correlation?

Answer/comment

The covariance of assets B and C is negative. We can see this simply by looking at the table and noting that in the states in which B “does well,” C “does poorly,” and vice versa.

Another way of seeing the same thing is noting that both B and D are individually risky (their payoffs are uncertain), but we just found in part 2.2 that a portfolio composed of one unit of asset B and two units of asset D is risk free. Essentially, the risk in each type of asset is *exactly offset* by the risk in the other. That is, B and C act as insurance for one another.

As we've talked about in class, the key feature of things that act as insurance is that they covary negatively with the thing they are insuring.

The correlation of assets B and C is also negative. As we've talked about in class, the sign of covariance and correlation are always the same. This follows from the definition of correlation:

$$\text{corr}(B, C) = \frac{\text{cov}(B, C)}{\sqrt{\text{var}(B)\text{var}(C)}}.$$

Importantly, note that the denominator is always positive, so the sign of the correlation is always the same as the sign of the covariance.

2.4 What is the variance of the price of asset D?

Answer/comment

Simply calculate the expected value (that is, the mean) of the price of asset D in period $t+1$ and then apply the variance formula.

$$\begin{aligned} E[P_{t+1}^C] &= .3(18) + .5(12) + .2(23) = 16 \\ \text{var}[P_{t+1}^C] &= .3(18-16)^2 + .5(12-16)^2 + .2(23-16)^2 \\ &= 19 \end{aligned}$$

That is, the answer is 19.

3 Analyzing a 10-year bond. Note: This problem uses the spreadsheet ps12014.xlsx provided with the problem set. Go to the tab in the spreadsheet labelled 'bond'. This spreadsheet allows you to enter a par value, coupon value, and constant interest rate i, and will use our standard formula to compute the present value and duration of the bond under the stated conditions.

Complete the spreadsheet following the instructions in blue. When you are done, as a check see that i) if you enter a bond with implied coupon rate equal to the specified i, then the present value equals the par value, and ii) if you put in a coupon of zero, the duration should equal 10, the time until to the only payment.

3.1 Take a bond with par value 200, coupon 5, and constant interest rate 2%. State the present value and duration.

Answer/comment
The present value is \$208.98 and the duration is 9 years.

3.2 According to the spreadsheet, what will be the present value of the bond at an interest rate of 2.5 percent?

Answer/comment
\$200.00. That is, when the coupon rate is equal to the yield to maturity, the present value equals the par value.

3.3 Given the change in the present value between part 3.2 and part 3.3, what is the implied rate of capital gain or loss?

Answer/comment
Just apply the formula for the (exact) rate of of capital gain:

$$\begin{aligned} RCG &= \frac{PV_1 - PV_0}{PV_0} \\ &= \frac{200 - 208.98}{208.98} \\ &= -.0430 \end{aligned}$$

The rate of capital gain is -4.30%.

3.4 Using our formula for the approximate rate of capital gain based on duration, if the interest rate rises to 2.5%, what will be the approximate rate of capital gain or loss?

Answer/comment

Just apply the formula for the approximate rate of capital gain:

$$\begin{aligned} RCG &\approx -DUR \times \frac{i_1 - i_0}{1 + i_0} \\ &= -9\frac{.025 - .02}{1 + .02} = -.0441 \end{aligned}$$

That is, the approximate rate of capital gain is −4.41%.

The formula used above can be found on page 59 of the textbook (8th edition).

Some of you wondered whether you should use the duration at the original interest rate or the duration at the second interest rate in this formula. The answer is that this formula is an approximation and these two different methods are simply two different approximations. Neither is 'right' in any deep sense.

In practice, when folks use the formula, they usually use the duration at i_0. Either will get full points here. Note that this answer is slightly different from the answer in part 3.3, although they are both getting at the same thing. That's because the formula used in 3.3 was exact, whereas the formula used in this part is an approximation. See the lecture notes for lecture 3 for more on this.

3.5 As the interest rate falls, what happens to the duration of this bond? Provide some intuition for this result.

Answer/comment

As the interest rate falls, the duration of the bond will rise.

As the interest rate falls, the present value of payments in the distant future rises relative to the present value of payments that arrive sooner. The further payments in the future are discounted relatively more at any given interest rate and this differential grows with the interest rate. Thus, at a lower interest rate a greater share of the present value of the bond comes farther into the future, meaning that the 'effective time' until you receive your present value is longer.

4 Some real world data. This problem uses the spreadsheet ps12015.xlsx provided with the problem set. Go to the tab in the spreadsheet labelled 'real returns'. This sheet gives annual real returns for the years 1954–2014 for 4 asset or portolios:

t1y... 1-year U.S. Treasury securities.
mkt... a broad portfolio of stocks
comps... a portfolio of computer related stocks
banks... a portfolio of banking related stocks

The Treasury data are from Fred (http://www.research.stlouisfed.org/fred2/) as are the CPI data used to compute inflation (in order to convert these to real returns.) The stock return data are from Ken French's data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The returns are transformed to approximate real returns by subtracting the CPI inflation rate.

4.1 Describe what sorts of firms are included in the 'comps' and 'banks' portfolios. (Hint: Go to the Ken French data library link above, scroll down to the 48 industry portfolios links and click on 'details', you should be able to get the answer from there.)

Answer/comment
The 'comps' portfolio is a portfolio of stocks in the computer industry, including office computers, mainframe, terminals, disk and tape drives, optical scanners, graphics, office automation systems, peripherals, equipment, magnetic and optical recording media, and computer integrated systems design. The 'banks' portfolio is made up of banking institutions, including depository institutions, commercial banks, federal reserve banks, and other types of banks and credit agencies.

The answers for this question can be found by clicking on the "Download industry definitions" link here.

4.2 Complete the spreadsheet by following the notes highlighted in blue. What are the mean and standard deviation of the returns on t1y, mkt, comps, and banks?

Answer/comment

mean:
t1y: 1.44%
mkt: 7.95%
fun: 13.20%
fin: 9.46%

std. dev.:
t1y: 2.17%
mkt: 18.59%
fun: 32.25%
fin: 24.22%

Note about calculating standard deviation: For reasons that are not central to our problem, when taking the standard deviation of a sample of data, one often divides the sum of squared deviations by $N - 1$ (one less than the number of terms in the sum) rather than N. In the spreadsheet, we have used the function `stdev`, which follows this convention. See the excel help for `stdev` vs. `stdevp` for a bit more on this, or consult any elementary statistics text. When N is largish, this obviously does not matter much.

4.3 Using the mean return, compute approximately how much money you would have at the end of 2014 if you had put $1 in the t1y asset (short-term Treasury security) at the beginning of 1954 and reinvested all proceeds each year?

Answer/comment

The mean return of the t1y asset is 1.44%. It is 61 years from the beginning of 1954 to the end of 2014. If I earn that return for 61 years, the value at the end of 2014 will be given by

$$\$1(1 + 0.0144)^{61} = \$2.39$$

Note. Although the question asks students to use mean returns, full credit will be given if students correctly use actual returns and compound them.

4.4 Using the mean return, compute approximately how much money you would have at the end of 2014 if you had put $1 in the mkt portfolio at at the beginning of 1954 and reinvested all proceeds each year?

Answer/comment
This question is similar to the previous one. The only difference is that now the mean return of mkt portfolio is 7.95%. So if I earn that return for 61 years, my value at the end of 2014 will be given by

$$\$1(1 + 0.0795)^{61} = \$106.31$$

4.5 What is the smallest rate of return you would have made in any year if you were invested in t1y? And the smallest returns for mkt and for banks?

Answer/comment
t1y: -2.96%
mkt: -38.34%
banks: -49.36%

Note: You could obtain these answers by visually searching each column for the smallest number, or, more efficiently, you could use the "MIN" function in Excel.

A general observation illustrated by these last few problems: The value of your savings over the long haul is tremendously affected by whether you put your funds in very safe or riskier investments. By the same token, if you put your money in risky assets such as those in this illustration, you face a substantial risk of losing, say, half your wealth over a very short period of time such as a year.

Thus, for example, if you are 30 years old and saving for retirement, putting your money in the stock market makes more sense than it does if you have $1,000 in savings and have a $1,050 bill coming due next year.

This perspective is enshrined in a very good book, 'Stocks for the long run' by Jeremy Siegel.

`http://en.wikipedia.org/wiki/Stocks_for_the_Long_Run`

4.6 You consider putting all your funds in a portfolio split between comps and banks. What would be the mean and standard deviation of the portfolio if you put the following percent of funds into comps: 10%, 25%, 50%, 75%, 90%?

Answer/comment

mean:
10%: 9.84%
25%: 10.40%
50%: 11.33%
75%: 12.26%
90%: 12.82%

stdDev:
10%: 23.45%
25%: 23.01%
50%: 24.21%
75%: 27.50%
90%: 30.22%

Along with problem 1.2, this problem was meant to illustrate that forming a portfolio can be thought of as creating a new asset with its own risk, return characteristics. In 1.2, we 'created' a risk free asset. Here, but varying the portfolio shares, we created a 'menu' of risk and return options that an investor could choose from.

In more advanced courses such as the investments course (180.367) you learn more about portfolio theory.