Problem set 1
607: Applied Macroeconometrics
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Fall 2015

The following is due at the beginning of next class. You can turn in any paper in my mailbox or in class or by email. You may work in groups; hand in a single submission for the group. The submission should list those who contributed.

1. Review. Review the 'Econometric and Statistical Basics' (linked with the problem set on website). For next week, define the following items from the basics:

   (a) Generalized least squares (GLS) estimator, feasible GLS estimator

   (b) Instrumental variables estimator
   Note: for each of these, give the textbook formula for the estimator for the slope coefficients and for the variance-covariance matrix of the estimates.

2. The DGP is an AR(1)

$$y_t = \mu + \rho y_{t-1} + \varepsilon_t$$

   $\varepsilon_t \sim iidN(0,1)$

   (a) Give expressions for the variance of y and the first two autocorrelations in terms of the parameters of the problem.

   (b) Suppose we have the value for an initial condition, $y_0 = \xi$. Write an expression for y_1 in terms of the initial condition and ε_1. Do the same for y_2, and then for any y_t, $t \geq 1$.

   (c) The DGP is the AR(1) described above. You estimate an AR(1) model by OLS:
   $$y_t = \alpha + \beta y_{t-1} + u_t$$
   and u_t is the residual. Is the OLS estimator of α and β unbiased? Why or why not?

(d) The DGP is the AR(1) described above. You estimate the following model by OLS:

$$y_t = \alpha + \beta\tau_t + u_t$$

where $\tau_t = t$, that is τ is a deterministic, linear time trend. And u_t is the residual.

Is the OLS estimator of α and β unbiased?

3. Continuing with the AR(1) expression from last problem.

If we set $\rho = 1$ and $\mu = 0$, the process for y_t is called a random walk (in this case, due to Gaussian innovations, we might more fully say a Gaussian random walk). With $\mu \neq 0$, it is a random walk with drift. For this problem, take the case of the Gaussian random walk without drift.

(a) What is the unconditional expectation of y_t (that is, you don't have y_0).

(b) What is the expectation of y_t, $t \geq 1$ conditional on $y_0 = \xi$?

(c) Suppose we have an initial condition, $y_0 = \xi$. Give an expression for the conditional variance of y_t given the initial condition. That is, give

$$E((y_t - \hat{y}_t)^2 | y_0 = \xi),$$

where

$$\hat{y}_t = E(y_t | y_0 = \xi)$$