Problem set 3
607: Applied Macroeconometrics
Fall 2016
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The following is due at the beginning of next class. You can turn in any paper in my mailbox or in class; email me and requested computer work. You may work in groups; hand in a single submission for the group. The submission should list those who contributed.

1. Define the following terms
   (a) Nominal size of a test
   (b) Power of a test
   (c) Neyman-Pearson Lemma
   (d) Define a consistent test
2. χ^2.
   (a) Define the χ^2 in terms of normals.
   (b) Sketch the $\chi^2_{(k)}$ density for $k = 1$. How does this differ from the density for higher k?
   (c) State the mean and variance of the $\chi^2_{(k)}$ as a function of k.
   (d) If $x \sim \chi^2_{(k)}$, what happens to the mean and variance of x/k as $k \to \infty$.
3. F. Suppose $x \sim F_{(k,\ell)}$.
   (a) Define the F distribution in terms of other familiar distributions.
   (b) For fixed $k = 1$, what does increasing ℓ do to the shape of the density? In the limit for large ℓ?
4. Take the AR(2) process,
$$A(L)y_t = \varepsilon_t$$
   where $A(L) = (1 - \rho_1 L - \rho_2 L^2)$. If the process is stationary, then we can also write the process as
$$y_t = B(L)\varepsilon_t$$

where $B(L) = A(L)^{-1}$, or

$$B(L)A(L) = 1$$

(a) Assuming $B(L) = \sum_{j=0}^{\infty} b_j L^j$ and using the definition of $A(L)$ and $B(L)A(L) = 1$ (and by matching coefficients on powers of L) find expressions for $b_0, \ldots, b_3$ in terms of ρ_1 and ρ_2.

(b) Give an expression for the variance of the process in terms of the ρs and variance of ε.

5. Take the MA(1) process:

$$y_t = (1 + \theta L)\varepsilon_t$$

with $\varepsilon_t \sim iidN(0, \sigma_\varepsilon^2)$.

Note: You may want to consult the lecture notes on covariance stationary math.

What is the variance of the process? And the first autocovariance?

(a) Now consider the process

$$y_t = (1 + \theta^{-1} L)\varepsilon_t$$

with $\varepsilon_t \sim iidN(0, \theta^2 \sigma_\varepsilon^2)$.
What is the variance of the process? And the first autocovariance?

(b) Using a sample of, say, 100 observations could you distinguish these processes?

(c) Suppose one of these two processes describes some real world process. Can it matter which is 'true'? Explain.

6. The class presentation.

Obtain monthly data for 3-month nominal interest rates in the US and UK and data for the dollar-pound exchange rate.

(a) Plot the interest rates and the log change $(ln(x_t) - ln(x_{t-1})$ in the exchange rate.

(b) When s_t is the log exchange rate and i_t and i_t^* are home and foreign 1-period interest rates, the uncovered interest rate parity relation is:

$$E_t s_{t+1} - s_t = i_t - i_t^*$$

Give an interpretation of this relation and explain why we might expect something like this relation to hold.

(c) To test the UIP relation, researchers often run a regression

$$s_{t+1} - s_t = \alpha + \beta(i_t - i_t^*) + \varepsilon_t$$

and test $\alpha = 0$ and $\beta = 1$. Under what assumptions about expectations is this a sensible test?

(d) What modifications/transformations to your data are needed to properly implement the test just described? Think about the following: Your interest rate data may be in annualized percent change. You have three-month rates measured at a monthly frequency. Exchange rate may be stated in dollars per pound or the other way. Would you rather use monthly data that is on an average-of-daily basis or end-of-month basis? Which do you have?

(e) Report results of the regression above for various sample periods in your data (e.g., whole sample period, split in the middle, etc.

(f) For the presenter: using Fama's article show how the above results have implications for the sign and variance of the risk premium.

cite: Fama 1984 E. Fama, Spot and forward exchange rates. Journal of Monetary Economics, 14 (1984), pp. 319–338.
http://dx.doi.org/10.1016/0304-3932(84)90046-1

note: Google `UIP puzzle` for many papers on this topic.