Problem set 8
ANSWERS
607: Applied Macroeconometrics
Fall 2016
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The following is due at the beginning of next class. You can turn in any paper in my mailbox or in class; email me and requested computer work. You may work in groups; hand in a single submission for the group. The submission should list those who contributed.

1. Define the following terms. For the latter two, your definition may be as formal or informal as you like, but I hope you cover the key points (which obviously requires a bit of judgment).

   (a) Random walk with drift.
   Answer/comment

   $$y_t = \alpha + y_{t-1} + \varepsilon_t$$

   where ε_t is mean zero and perhaps iid.

   (b) Standard Brownian Motion (Weiner process)
   Answer/comment
   See the lecture notes, I just wanted you to have to write it out once.

   (c) Donsker's Theorem
   Answer/comment
   I'v never asked this before, so I'm interested to see your answers. If you google around, you'll see that this is pretty deep. The part I hope you figured out is something like this:
   Suppose $\varepsilon_t \sim iid$ and mean zero and variance 1, and $X_T = (x_1, \ldots, x_T)$ defined by:

   $$x_t = \sum_{t=1}^{t} \varepsilon_t$$

Then there is a way of reconceptualizing X_T (the sample) as a function, $\tilde{X}_T(r)$, on $r \in [0,1]$ where the transformation neither adds nor subtracts information, and

$$\tilde{X}_T(r) \rightarrow_d W(r)$$

where $W(r)$ is a standard Brownian motion, or Weiner process. Thus, in a colloquial sense, discrete-time random walks are asymptotically equivalent to Weiner processes. (And a whole bunch of measure theory goes by in that term *asymptotically equivalent*, which I am not using here in any formal or standard way.)

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2. Persistence in the AR(1) case. The DGP is a univariate Gaussian AR(1):

$$y_t = \alpha + \rho y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim iidN(0,1)$$

Assume $y_0 = \xi$. Your sample goes from $t = 1, \ldots, T$.

For $|\rho| < 1$, the OLS $\hat{\rho}$ based on the observations $2, \ldots, T$ is in the CAN framework.

(a) For various T, ρ, with $\alpha = \xi = 0$, run a Monte Carlo. Present a matrix of histograms for $\hat{\rho}$ and for the t-statistic for testing ρ equal to the true ρ in the DGP.

Answer/comment

The lectures are all about this Monte Carlo. I hope you find that the lectures enable you to see things in the results that otherwise would have been a bit of a jumble.

(b) Repeat the previous but with no constant in the regression—that is, imposing the true value of α.

(c) Describe the results.

3. Suppose x and y are independent Gaussian random walks with initial conditions $x_0 = y_0 = 0$. Consider the regression of

$$y_t = \beta x_t + \varepsilon_t$$

With the two random walks independent of each other, the 'true' β is $\beta^* = 0$.

(a) Is the OLS $\hat{\beta}$ consistent for β^*?

(b) Run a Monte Carlo to investigate this regression. For various T, give histograms for a) $\hat{\beta}$, b) $\sqrt{T}(\hat{\beta} - \beta^*)$, and the R^2 for this regression.