Problem set 9
607: Applied Macroeconometrics
Fall 2016
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The following is due at the beginning of next class. You can turn in any paper in my mailbox or in class; email me and requested computer work. You may work in groups; hand in a single submission for the group. The submission should list those who contributed.

1. Terms. The latter to may be as formal or informal as you like, but I hope you cover the key points (which obviously requires a bit of judgment).
   (a) Random walk with drift.
   (b) Standard Brownian Motion (Weiner process)
   (c) Donsker's Theorem (as formal or informal as you like)

2. Persistence in the AR(1) case. The DGP is a univariate Gaussian AR(1):

$$y_t = \alpha + \rho y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim iidN(0,1)$$

   Assume $y_0 = \xi$. Your sample goes from $t = 1, \ldots, T$.

   For $|rho| < 1$, the OLS $\hat{\rho}$ based on the observations $2, \ldots, T$ is in the CAN framework.

   (a) For various T, ρ, with $\alpha = \xi = 0$, run a Monte Carlo. Present a matrix of histograms for $\hat{\rho}$ and for the t-statistic for testing ρ equal to the true ρ in the DGP.
   (b) Repeat the previous but with no constant in the regression—that is, imposing the true value of α.
   (c) Describe the results.

3. Suppose x and y are independent Gaussian random walks with initial conditions $x_0 = y_0 = 0$. Consider the regression of

$$y_t = \beta x_t + \varepsilon_t$$

With the two random walks independent of each other, the 'true' β is $\beta^* = 0$.

(a) Is the OLS $\hat{\beta}$ consistent for β^*?

(b) Run a Monte Carlo to investigate this regression. For various T, give histograms for a) $\hat{\beta}$, b) $\sqrt{T}(\hat{\beta} = \beta^*)$, and the R^2 for this regression.